Appendix: Kin-Cohort Measures of Maternal Cumulative Prevalence of Infant (mIM), Under-five (mU5M), and Offspring Mortality (mOM)

In this appendix, we provide technical details of the indirect estimation method that we developed for computing the three indicators presented in the main text: the maternal cumulative prevalence of infant (mIM), under-five (mU5M), and all offspring mortality (mOM).

1. **Cumulative offspring death over a woman's life-course by offspring's age at death**

Our estimates are based on a set of mathematical relationships known as the Goodman-Keyfitz-Pullum kinship equations (GKP equations).[1] The GKP equations state that, in a population closed to migration where mortality and fertility remain constant over time—i.e., a demographically stable population—the number of children surviving to a woman aged *a* must be:

$$CS_a = \int_{\alpha}^{\beta} {}_1F_x l_{a-x}\, dx \qquad (1)$$

where m_x is the fertility rate at age *x* and l_{a-x} are the survival probabilities until age (a-x). Recent work has shown that it is possible to extend the GKP equations to real-world populations where mortality and fertility vary over time.[2] Of relevance to this study is the notion that empirical life tables and fertility data can be used to estimate the cumulative number of offspring deaths, *OD(a,c)*, experienced by an average woman born in year *c* surviving to age *a* in a non-stable population:

$$OD(a,c) = \sum_{x=15}^{a} {}_1F_x(c) - \sum_{x=15}^{a} [\, {}_1F_x(c) \quad l_{a-x}(c+x)]. \qquad (2)$$

Here, ${}_1F_x(c)$ are single-age age-specific fertility rates at age *x* for women born in cohort *c* and $l_{a-x}(c+x)$ are survival probabilities until age *(a-x)* for women born in cohort *(c+x)*. Cohort demographic rates are not readily available for all world countries. After calculating cohort-estimates [2], we extended the models to approximate single-age and single-year cohort fertility and survival rates using demographic period data from the 2019 Revision of the United Nations World Population Prospects (UN WPP).

Note that Equation (2) refers to all-age offspring deaths, but our study required us to distinguish between all-age offspring deaths (mOM) and those to children younger than age five (mU5M) and to children younger than age one (mIM). We estimate the cumulative number of offspring deaths experienced by a woman surviving to age *a* according to the offspring's age at death as:

$$OD^k(a,c) = \sum_{x=15}^{a} {}_1F_x(c) - \sum_{x=15}^{a} [\, {}_1F_x(c) \quad l_{(\min(a-x,k))}(c+x)] \qquad (3)$$

where $k = 1$ for infant deaths, $k = 5$ for child deaths and $k = 100$ for all-age offspring deaths. We restrict the female reproductive age $[\alpha, \beta]$ to $(\alpha, \beta, n) = (15{,}49{,}1)$, so that $a \leq \beta + k$ for all cases.

2. Proportion of bereaved mothers per 1,000 mothers

We generate our estimates in four steps. First, we determine the prevalence of bereaved women in a population by considering the probability that an average woman will experience the death of a child when she is *a* years old ${}_1q_a^k(c) = 1 - e^{-h(a,c)}$, where $h(a,c) = OD^k(a+1,c) - OD^k(a,c)$ is the hazard rate of experiencing the death of a child younger than *k*. In demography, the life-table columns (${}_nq_a$, l_x) and hazard rates usually refer to an individual's own death, but here they mean something slightly different. For example, ${}_1q_a^k$ does not refer to the probability of death for a person, but the probability that a women experiences the death of a child. Following standard demographic methods[3] we create a life table with a unit radix $l_0^k = 1$ where ${}_1q_a^k(c)$ is the probability of losing a child between ages *a* and *a+1*. We define $FOD^k(a,c) = 1 - l_a^k(c)$ as the fraction of women aged *a* in cohort *c* who ever experienced the death of at least one child younger than *k* years.

Second, we account for the mortality of women by considering $\mathrm{FWS}(a,c)$, the fraction of women that survived up to age *a* after the start of reproductive age ($\alpha = 15$) in each birth cohort (where $a > \alpha$). We approximate this value using country-specific period life tables from the UN WPP. The proportion of women (per 1,000 women) who have ever lost one or more children younger than *k* is $wOM^k(a,c) = FOD^k(a,c) * FWS(a,c) * 1000$. Note that these estimates pertain to all women in a population, including those who have never had a live birth.

Third, we estimate an equivalent measure for mothers by rescaling our estimates using a similar life table approach to the one discussed above. We consider fertility as a "hazard rate" to approximate the number of women that "survive" having children (i.e., remain childless) after experiencing a set of age-specific fertility rates. The fraction of women who have ever been mothers $FM(a,c)$ is approximated as 1 minus the fraction of childless women. We can now define, for a given cohort, the proportion of mothers (per 1,000 mothers) who have ever lost one or more children younger than *k*:

$$mOM^k(a,c) = wOM^k(a,c) * FM(a,c). \tag{4}$$

Fourth, we generate period estimates of the prevalence of bereaved mothers, comparable to the empirical survey-based estimates, using different combinations of cohort and age. Equation (4) produces single age estimates of the prevalence of maternal bereavement. We grouped these by maternal ages 20-44 and 45-49 to match the survey estimates in the main text (**figures 1-5** and **table s1**) and by 5-year age groups to produce the results displayed in **table S2**.

References

1. Goodman, L. A. Family Formation and the Frequency of Various Kinship Relationships. *Theor. Popul. Biol.* 27 (1974) doi:10.1016/0040-5809(74)90049-5.
2. Alburez-Gutierrez, D., Kolk, M. & Zagheni, E. Women's experience of child death over the life course: A global demographic perspective. *Demography* (Forthcoming) doi:10.31235/osf.io/s69fz.
3. Preston, S. H., Heuveline, P. & Guillot, M. *Demography: measuring and modeling population processes*. (Blackwell Publishers, 2001).